Advanced Probability, MA 687-OP Summer 2026

Class meets MWF, 9:40–11:00, UH 4004

Instructor: Dr. Nandor Simanyi
Office: UH 4014, ph. 934-2154
Office hours: MW, 11:00–12:00, or by appointment
E-mail: simanyi@uab.edu

Prerequisites: Successful completion of the Real Analysis I and II courses.

Text: Patrick Billingsley: Probability and Measure, 3rd Ed. Wiley Series in Probability and Mathematical Statistics, Chapters 4–6.

Additional text: Electronic material, including homework assignments and handouts, will be available on Canvas and at http://people.cas.uab.edu/~simanyi/teaching/MA-687/

Grading policy:

Homework	30 %
Midterm Exam: Mid-July,	30 %
Final Exam, ...	40 %

Homework: The list of homework exercises and due dates will be posted on the instructor's web page and on Canvas. You need to submit your homework, on paper, on the due date in class. No late homework. Exercises marked as "Bonus" can be attempted for extra credit.
Tests: Proofs are required. The final test is comprehensive.

Overall Goals: The main purpose of this course is to make PhD students familiar with the analytic concepts of probability theory, based upon the notions, concepts and facts from Lebesgue measure and integral theory. This includes, but not restricted to, the notions of random variable, expected value, moments of random variables, distributions, continuity and absolute continuity of distributions, conditional probability and distribution, cumulative distribution function, density function, convergence of distributions, weak convergence, Fourier transform of distributions (i.e. the characteristic function), the Moment Generating Function, martingales, and efficiently use these notions to prove fundamental results like the Weak and Strong Law of Large Numbers, the Central Limit Theorem, and martingale convergence theorems.

Tentative content:

1. Review of basic concepts of measure and integral theory: Probability spaces, integral, absolute continuity, Radon-Nikodym derivative, conditional expectation, L^p-spaces,

almost sure and L^p convergence, stochastic convergence, convergence in distributions. Independence and products of measure spaces.

2. Random variables, their distributions and expected values.
3. Sums of independent random variables.
4. Poisson processes.
5. The Von Neumann and the Birkhoff Ergodic Theorem.
6. Simple stochastic processes: Finite state Markov-chains, ergodic Markov-chains.
7. The Weak and Strong Law of Large Numbers.
8. Fourier transform and the Central Limit Theorem.
9. The Method of Moments.
10. Martingales, martingale convergence theorems. (Time permitting.)

Welcome to my MA 687 class, and best of luck to you all!